education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

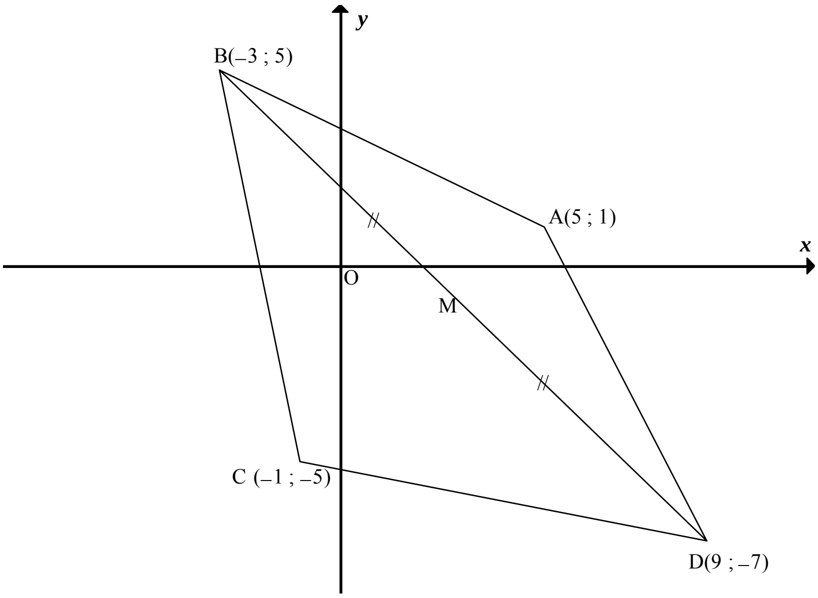
NOVEMBER 2009

MEMORANDUM

MARKS: 150

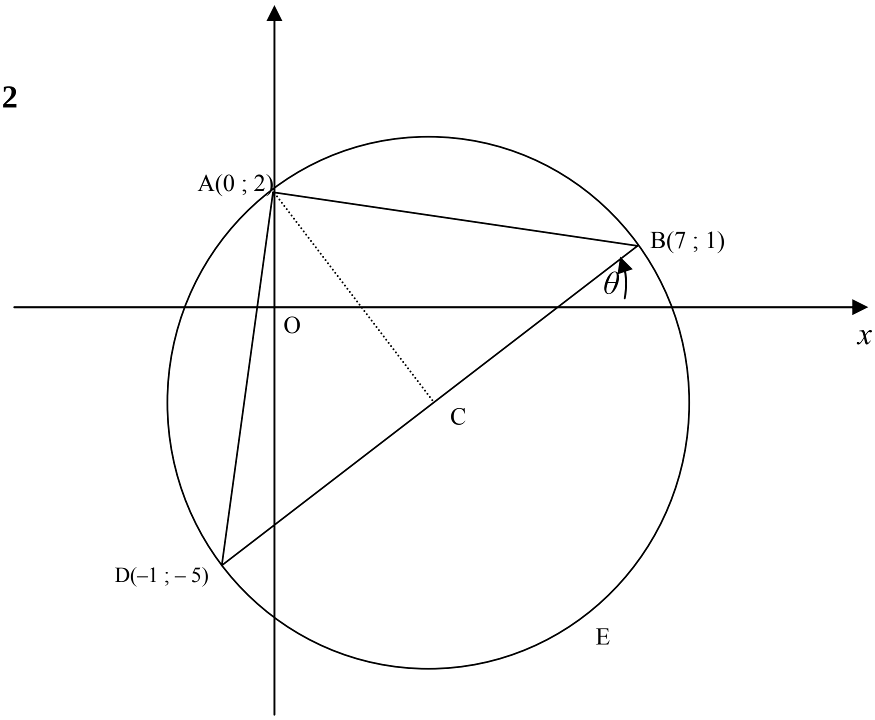
This memorandum consists of 15 pages.

QUESTION 1



1.1	$m_{AC} = \frac{1-(-5)}{5-(-1)}$ $m_{AC} = 1$	✓ substitution into gradient formula ✓ answer (2)
1.2	Equation of AC is: $y - 1 = 1(x-5)$ $y = x - 4$	✓✓ substitution into straight line equation ✓ answer (3)
1.3	$\mathrm{M}(x;y) = \left(\frac{9-3}{2};\frac{-7+5}{2}\right)$ $= (3\,;-1)$ Substitute $x = 3$ into equation of AC: $y = 3 - 4 = -1$ $\therefore$ M lies on line AC	✓ midpoint ✓substitution of $x = 3$ ✓$y = -1$ (3)
1.4	$m_{BM} = \frac{5+1}{-3-3}$ $m_{BM} = -1$ $m_{AC} = 1$ $m_{BM} . m_{AC} = -1 \times 1$ $= -1$ $\therefore BM \perp AC$ or $\mathrm{A\hat{M}B} = 90°$	✓ substitution into gradient formula ✓ $m_{BM} = -1$ ✓ $m_{BM} \times m_{AC} = -1$ (3)
1.5	$\mathrm{BM} = \sqrt{(5+1)^2 + (-3-3)^2}$ $\mathrm{BM} = \sqrt{72}$ $\mathrm{AC} = \sqrt{(5+1)^2 + (1+5)^2}$ $\mathrm{AC} = \sqrt{72}$ Area of $\Delta ABC = \frac{1}{2}\left(\sqrt{72}\right)\left(\sqrt{72}\right)$ = 36 square units	✓ substitution into distance formula ✓BM= $\sqrt{72}$ ✓AC= $\sqrt{72}$ ✓ formula for area of Δ ✓ answer (5) **[16]**

QUESTION 2



2.1.1	Midpoint BD : $\left(\frac{7-1}{2};\frac{1-5}{2}\right)$ $= (3; -2)$	✓substitution into midpoint formula ✓answer (2)
2.1.2	$CA=\sqrt{(3-0)^2+(-2-2)^2}=\sqrt{25}=5$ $CB=\sqrt{(3-7)^2+(-2-1)^2}=\sqrt{25}=5$ $\therefore CA=CB=CD$	✓CA ✓CD ✓CB (3)
2.1.3	$r = 5$ and centre$(3 ; -2)$ Equation of circle is $(x-3)^2+(y+2)^2=25$	✓radius and centre ✓equation of circle (2)
2.1.4	$m_{BD}=\frac{1-(-5)}{7-(-1)}$ $=\frac{3}{4}$ $m_{BD}=\tan\theta=\frac{3}{4}$ $\therefore \theta = 36{,}87°$	✓gradient $\frac{3}{4}$ ✓ $\tan\theta=\frac{3}{4}$ ✓ answer (3)
2.1.5	Let E (x ; y) $\frac{x+0}{2}=3$ $\frac{y+2}{2}=-2$ $\therefore x=6$ $\therefore y=-6$ E(6 ; −6) **OR** E(3 + 3; −2 − 4) = (6 ; −6)	✓$x = 6$ ✓$y = -6$ (2) **OR** ✓✓ with transformation (2)

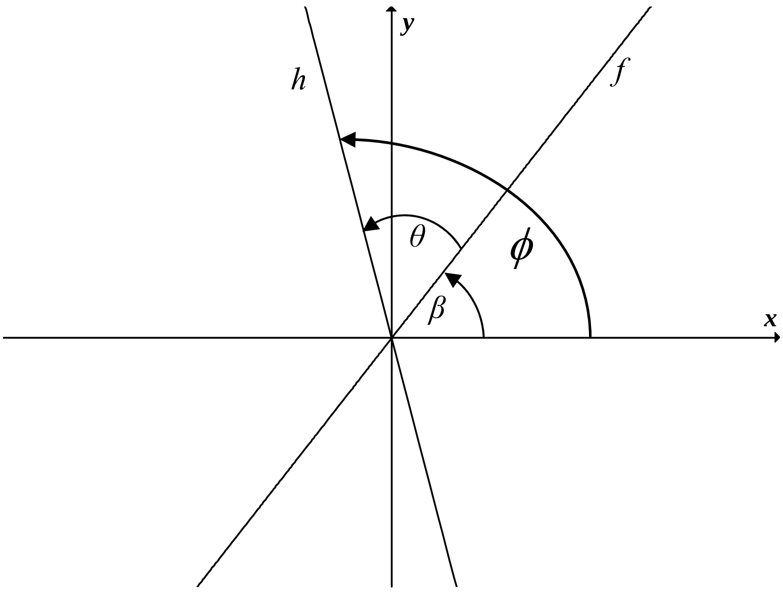
2.1.6	The diagonals AE and BD bisect each other $m_{AB} \times m_{AD} = \frac{1-2}{7-0} \times \frac{2+5}{0+1} = \frac{-1}{7} \times 7 = -1$ $\therefore \hat{A} = 90°$ $\therefore$ ABED is a rectangle (diagonals bisect each other and adjacent sides are perpendicular) **OR** Show that $\hat{A} = \hat{B} = \hat{C} = \hat{D} = 90°$	✓ bisect (from 2.1.2) ✓ gradients ✓ 90° (3)
2.1.7	Gradient of tangent is = $-\frac{4}{3}$. Equation of a tangent at B is: $y - 1 = -\frac{4}{3}(x-7)$ $y = -\frac{4}{3}x + \frac{28}{3} + 1$ $y = -\frac{4}{3}x + \frac{31}{3}$ **OR** $y - 6 = -\frac{4}{3}(x)$ $1 = -\frac{4}{3}(7) + c$ $c = \frac{31}{3}$ $\therefore y = -\frac{4}{3}x + \frac{31}{3}$	✓gradient $-\frac{4}{3}$ ✓substitution into straight line formula ✓ answer (3) **OR** ✓gradient $-\frac{4}{3}$ ✓ substitution into straight line formula ✓ answer (3)
2.2.1	$x^2 + 2x + \left(\frac{1}{2}(2)\right)^2 + y^2 - 4y + \left(\frac{1}{2}(-4)\right)^2 = 5 + 1 + 4$ $(x+1)^2 + (y-2)^2 = 10$ But $(x;y) \rightarrow (x-2; y+4)$ $\therefore (-1\,;2) \rightarrow (-1-2\,;2+4)$ $= (-3\,;6)$ $\therefore (x+3)^2 + (y-6)^2 = 10$ **OR** $(x+2)^2 + 2(x+2) + (y-4)^2 - 4(y-4) - 5 = 0$ $x^2 + y^2 + 6x - 12y + 35 = 0$ $(x+3)^2 + (y-6)^2 = 10$	✓completing the square ✓ equation ✓ equation (3) **OR** ✓replacing x with $x + 2$ and replacing y with $y - 4$. ✓ completing the square ✓ equation (3)

2.2.2	Distance from origin to centre $= \sqrt{(-3-0)^2+(6-0)^2} = \sqrt{45}$ Since $\sqrt{45} > \sqrt{10}$, the origin lies outside the circle.	✓ distance from origin to centre ✓ conclusion (2) **[23]**

QUESTION 3

3.1	Enlargement with a scale factor of 2	✓ enlargement ✓ scale factor 2 (2)
3.2	$R(-1\,;3) \rightarrow R^{//}(-1+3\ ;3+4) = R^{//}(2\,;7)$	✓2 ✓7 (2)
3.3	$(x\,;y) \rightarrow (y\,;x)$ **OR** Reflection about the line $y = x$	✓✓ $(y\,;x)$ (2) **OR** ✓reflection ✓ line $y = x$ (2)
3.4	$P(-5\,;2) \rightarrow P^{/////}(-2\,;-5)$ $(x\,;y) \rightarrow (-y\,;x)$ $\therefore \theta = 90°$… rotation in an anticlockwise direction **OR** $P(-5\,;2) \rightarrow P^{/////}(-2\,;-5)$ $(x\,;y) \rightarrow (-y\,;x)$ $\therefore \theta = 270°$... rotation in a clockwise direction	✓✓for any new correct coordinate ✓ 90° ✓ anti-clockwise (4) **OR** ✓✓for any new correct coordinate ✓ 270° ✓ clockwise (4) **[10]**

QUESTION 4

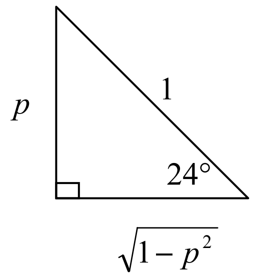
4.1		
	$\tan(\phi) = -6$ $\phi = 180° - 80{,}53767...°$ $\phi = 99{,}46°$ $\tan\beta = 2$ $\beta = 63{,}43°$ $\theta = \phi - \beta = 99{,}46° - 63{,}43°$ $\theta = 36{,}03°$	✓ substitution ✓ substitution ✓ method ✓ answer (4)
	OR	**OR**
	NOTE: The following solution is beyond the bounds of the curriculum, but is mathematically correct. $\tan\theta = \tan(\phi - \beta)$ $\tan\theta = \dfrac{\tan\phi - \tan\beta}{1 + \tan\phi.\tan\beta}$ $\tan\theta = \dfrac{-6-2}{1+(-6)(2)} = \dfrac{-8}{-11} = \dfrac{8}{11}$ $\therefore\ \theta = 36{,}03°$	✓ formula ✓ expansion ✓ simplification ✓ answer (4)

4.2	For clockwise rotation: $x^{/} = x\cos\theta + y\sin\theta$ $x^{/} = 3\cos 150° - 4\sin 150°$ $x^{/} = \frac{-3\sqrt{3}}{2} - 2$ $y^{/} = -x\sin\theta + y\cos\theta$ $y^{/} = -3\sin 150° - 4\cos 150°$ $y^{/} = \frac{-3}{2} + 2\sqrt{3}$ $P^{/}\left(-\frac{3\sqrt{3}}{2} - 2\,;\, -\frac{3}{2} + 2\sqrt{3}\right)$ $P^{//}\left(\frac{3\sqrt{3}}{2} + 2\,;\, -\frac{3}{2} + 2\sqrt{3}\right)$	✓ formula ✓ substitution ✓ simplification ✓ answer ✓ formula ✓ answer ✓ answer ✓ answer (8)
	OR	**OR**
	For anticlockwise rotation: $x' = x\cos\theta - y\sin\theta$ $= 3\cos(-150°) - (-4)\sin(-150°)$ $= 3(-\cos 30°) - 4\sin 30°$ $= 3\left(-\frac{\sqrt{3}}{2}\right) - 4\left(\frac{1}{2}\right)$ $= -\frac{3\sqrt{3}}{2} - 2$ $y' = x\sin\theta + y\cos\theta$ $= 3\sin(-150°) + (-4)\cos(-150°)$ $= -3\sin 30° - 4(-\cos 30°)$ $= -\frac{3}{2} + \frac{4\sqrt{3}}{2}$ $= -\frac{3}{2} + 2\sqrt{3}$ $P^{/}\left(-\frac{3\sqrt{3}}{2} - 2\,;\, -\frac{3}{2} + 2\sqrt{3}\right)$ $P^{//}\left(\frac{3\sqrt{3}}{2} + 2\,;\, -\frac{3}{2} + 2\sqrt{3}\right)$	✓ formula ✓ substitution ✓ simplification ✓ answer ✓ formula ✓ answer ✓ answer ✓ answer (8) **[12]**

QUESTION 5

5.1.1	$\tan\theta = -\frac{5}{12}$	✓ answer (1)
5.1.2	$r^2 = (-12)^2 + (5)^2 = 169$ $\therefore r = 13$ $\cos\theta\sin\theta = \frac{-12}{13} \times \frac{5}{13} = -\frac{60}{169}$	✓ r = 13 ✓ $\frac{-12}{13}$ ✓ $\frac{5}{12}$ ✓ answer (4)
5.2	$\frac{\sin(90° - x)\tan(360° - x)}{\cos(180° - x)}$ $= \frac{(\cos x)(-\tan x)}{-\cos x}$ $= \tan x$	✓ $\cos x$ ✓ $-\tan x$ ✓ $-\cos x$ ✓ answer (4)
5.3	$\frac{\cos(-60°) + \tan 135°}{\tan 315° + \cos 660°}$ $= \frac{\cos 60° - \tan 45°}{-\tan 45° + \cos 60°}$ $= \frac{\frac{1}{2} - 1}{-1 + \frac{1}{2}}$ $= 1$	✓simplification denominator ✓simplification numerator ✓ substitution ✓ answer (4)
5.4	$\frac{1}{2}\sin x = -0{,}243$ $\sin x = -0{,}486$ Ref angle = 29,078...° $\therefore x = 180° + 29{,}078...° + k.360°$ or $x = 360° - 29{,}078...° + k.360°$ $x = 209{,}08° + k.360°, k \in Z$ or $x = 330{,}92° + k.360°, k \in Z$ **OR** $x = k.360° - 150{,}92°$ or $x = k.360° - 29{,}08°$	✓-0,486 ✓ ref angle ✓ $209{,}08° + k.360°$ ✓ $330{,}92° + k.360°$ ✓ $k \in Z$ (5)
5.5	$\frac{\sin x}{\cos x} = \frac{\sqrt{3}\sin x}{\sin x}$ $\tan x = \sqrt{3}$ $x = 60°$ or $x = 240°$	✓equation ✓simplification ✓ answer (any one) (3) **[21]**

QUESTION 6

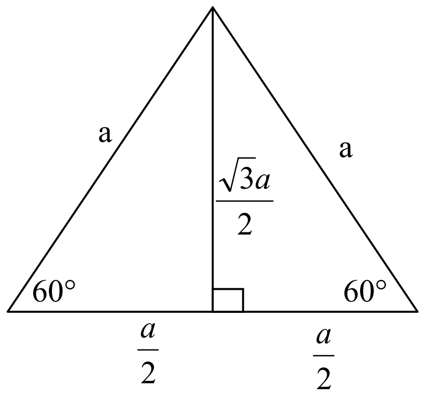
6.1.1	$\cos 24° = \dfrac{\sqrt{1-p^2}}{1} = \sqrt{1-p^2}$ 	✓diagram ✓ answer (2)
6.1.2	$\sin 12° \cos 12° - \sin(-66°)\tan 204°$ $= \dfrac{1}{2}(2\sin 12° \cos 12°) + \sin 66°(\tan 24°)$ $= \dfrac{1}{2}\sin 24° + \cos 24° \tan 24°$ $= \dfrac{p}{2} + \left(\dfrac{\sqrt{1-p^2}}{1}\right)\left(\dfrac{p}{\sqrt{1-p^2}}\right)$ $= \dfrac{p}{2} + p$ $= \dfrac{3p}{2}$ **OR** $\sin 12° \cos 12° - \sin(-66°)\tan 204°$ $= \dfrac{1}{2}\sin 24° + \cos 24° \tan 24°$ $= \dfrac{1}{2}\sin 24° + \sin 24°$ $= \dfrac{3}{2}\sin 24°$ $= \dfrac{3p}{2}$	✓ $\dfrac{1}{2} 2\sin 12° \cos 12°$ ✓ $\tan 24°$ ✓ $\sin 24°$ ✓ $\cos 24°$ ✓substitution ✓ answer (6) **OR** ✓ $\dfrac{1}{2}\sin 24°$ ✓ ✓ $\cos 24° \tan 24°$ ✓ $\sin 24°$ ✓simplification ✓ answer (6)
6.2	$LHS = \dfrac{2\sin x + \sin 2x}{4 + 3\cos x - \cos 2x}$ $= \dfrac{2\sin x + 2\sin x \cos x}{4 + 3\cos x - 2\cos^2 x + 1}$ $= \dfrac{2\sin x(1+\cos x)}{5 + 3\cos x - 2\cos^2 x}$ $= \dfrac{2\sin x(1+\cos x)}{(5 - 2\cos x)(1+\cos x)}$ $= \dfrac{2\sin x}{5 - 2\cos x}$	✓ $\cos 2x = 2\cos^2 x - 1$ ✓ $\sin 2x = 2\sin x.\cos x$ ✓ common factor ✓factors (4) **[12]**

QUESTION 7

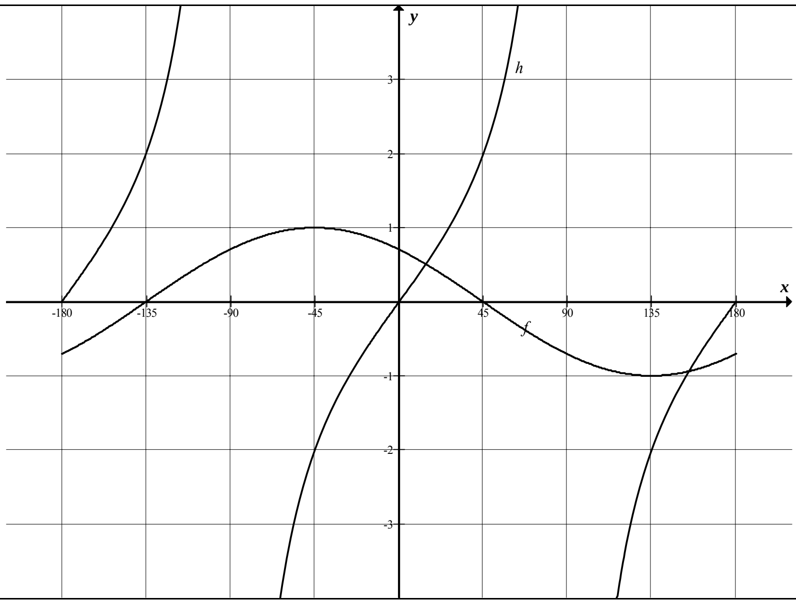
7.1.1	$Q\hat{R}S = 180° - (30° + 150° - \alpha)$ (3 angles of triangle) $= \alpha$ In triangle QRS: $\dfrac{QR}{\sin(150° - \alpha)} = \dfrac{12}{\sin\alpha}$ $QR = \dfrac{12\sin(150° - \alpha)}{\sin\alpha}$ $= \dfrac{12(\sin 150°\cos\alpha - \cos 150°\sin\alpha)}{\sin\alpha}$ $= \dfrac{12\left(\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right)}{\sin\alpha}$ $= \dfrac{6(\cos\alpha + \sqrt{3}\sin\alpha)}{\sin\alpha}$	✓ $Q\hat{R}S = \alpha$ ✓ substitution into sine rule ✓ rewriting in terms of QR ✓ expansion ✓ answer (5)
7.1.2	$\dfrac{PQ}{QR} = \tan\alpha$ $PQ = \left(\dfrac{6(\cos\alpha + \sqrt{3}\sin\alpha)}{\sin\alpha}\right)\tan\alpha$ $PQ = \left(\dfrac{6\cos\alpha + 6\sqrt{3}\sin\alpha}{\sin\alpha}\right)\dfrac{\sin\alpha}{\cos\alpha}$ $PQ = \dfrac{6\cos\alpha + 6\sqrt{3}\sin\alpha}{\cos\alpha}$ $PQ = \dfrac{6\cos\alpha}{\cos\alpha} + \dfrac{6\sqrt{3}\sin\alpha}{\cos\alpha}$ $PQ = 6 + 6\sqrt{3}.\tan\alpha$ **OR**	✓ ratio ✓ substitution of QR ✓ $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ ✓ simplification ✓ simplification (5) **OR**

	In triangle QPR: $Q\hat{P}R = 180° - (90° + \alpha) = 90° - \alpha$ (3 angles triangle) $\frac{PQ}{\sin\alpha} = \frac{QR}{\sin(90° - \alpha)}$ $PQ = \frac{QR\sin\alpha}{\cos\alpha}$ $= QR\tan\alpha$ $= \frac{12\sin(150° - \alpha)\tan\alpha}{\sin\alpha}$ $= \frac{12(\sin 150°\cos\alpha - \cos 150°\sin\alpha)\tan\alpha}{\sin\alpha}$ $= \frac{12\left(\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right)}{\sin\alpha} \times \frac{\sin\alpha}{\cos\alpha}$ $= 12\left(\frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sin\alpha}{\cos\alpha}\right)$ $= 6 + 6\sqrt{3}\tan\alpha$ $= 6(1 + \sqrt{3}\tan\alpha)$	✓ PQ = QRtanα ✓ substitution of QR ✓ simplification ✓ $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ ✓ simplification (5)

7.1.3	$6 + 6\sqrt{3}.\tan\alpha) = 23$ $6\sqrt{3}.\tan\alpha = 17$ $\tan\alpha = \frac{17}{6\sqrt{3}}$ $\tan\alpha = 1{,}635825763...$ $\alpha = 58{,}56°$	✓ equating ✓ simplification to $\tan\alpha = \frac{17}{6\sqrt{3}}$ ✓ answer (3)
7.2	Area of red part = Area of bigger triangle – area smaller triangle $= \frac{1}{2}.(80)(80).\sin 60° - \frac{1}{2}(50)(50).\sin 60°$ $= 1\,688{,}75 \text{ cm}^2$ **OR**	✓ method ✓ area formula ✓ substitution ✓ answer (4) **OR**

		
	Area of equilateral triangle of side $a = \frac{1}{2}.a.\frac{\sqrt{3}q}{2} = \frac{\sqrt{3}}{4}a^2$ $\therefore \frac{\sqrt{3}}{4}(80^2 - 50^2) = 975\sqrt{3}\ cm^2$	✓ method ✓ area formula ✓ substitution ✓ answer (4)
	OR	**OR**
	Area of red part = $\frac{\sqrt{3}}{4}(80^2 - 50^2)$ $= 975\sqrt{3}$ = 1 688,75 cm^2	✓ method ✓ area formula ✓ substitution ✓ answer (4) **[17]**

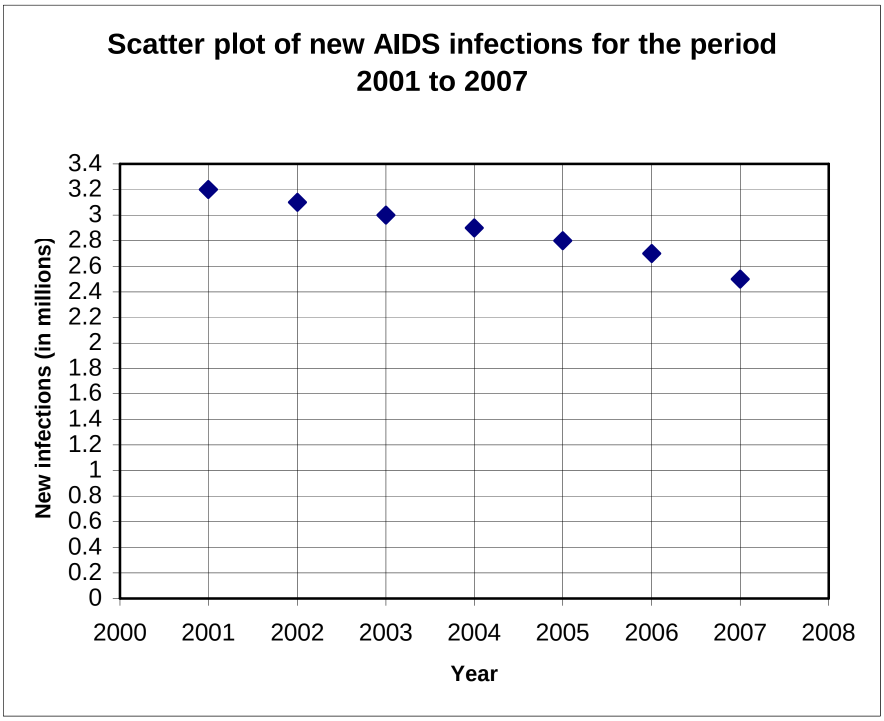
QUESTION 8

<table>
<tr><td>8.1</td><td>$a \tan 45° = 2$
a = 2</td><td>✓ reading from graph
✓ answer (2)</td></tr>
<tr><td>8.2</td><td>
</td><td>✓ amplitude
✓ shape
✓ endpoints
✓x-intercepts
(4)</td></tr>
<tr><td>8.3</td><td>2</td><td>✓ answer
(1)</td></tr>
<tr><td>8.4</td><td>$f(14{,}5°) = \cos 59{,}5° = 0{,}5075$
$h(14{,}5°) = 2\tan 14{,}5° = 0{,}5172 \ > \ f(14{,}5°)$

From the graph, 14,5° lies to the RIGHT of the point of intersection.
$\theta < 14{,}5°$</td><td>✓ calculation of $f(14{,}5°)$ and $h(14{,}5°)$

✓ $h(14{,}5°) > f(14{,}5°)$
✓ $\theta < 14{,}5°$
(3)
[10]</td></tr>
</table>

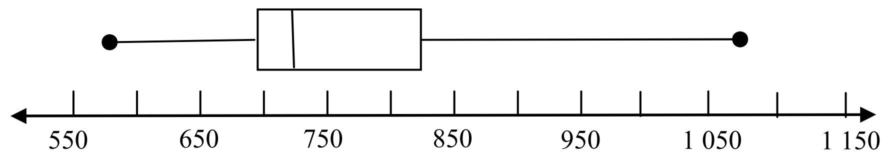
QUESTION 9

9.1		✓labelling of axes and title ✓plotting points ✓ labels (3)
9.2	There is more or less a linear decrease in the number of new infections year after year.	✓ decrease (1)
9.3	There is greater awareness about the AIDS pandemic through advertising in the media. **OR** Education in lifestyle has happened with many people. **OR** Any valid reason in the context of the situation	✓ ✓two valid reasons (2) **[6]**

QUESTION 10

10.1	$\bar{x} = 26$ *(Calculator used)*	✓✓ answer (2)
10.2	$\sigma = 8{,}336666...$ $\sigma = 8{,}34$	✓✓ answer (2)
10.3	The times are more closely spread around the mean because 13 out of 20 travelling times from the data set falls within one standard deviation from the mean. So the teacher's observation is acceptable.	✓comment on spread ✓use of SD in explanation (2) **[6]**

QUESTION 11

11.1	575 598 599 667 691 701 701 701 704 716 724 747 747 764 825 891 946 996 1 070 Median = 716 Lower quartile = 691 Upper quartile = 825	✓ arranging in ascending order ✓ median ✓ lower quartile ✓ upper quartile (4)
11.2	**Petrol**  	✓ box ✓ whiskers (2)
11.3	There are 19 data points. The lower quartile (600) is at position 5 and the upper quartile (800) is at position 15. There are 11 data points from 600 to 800. Therefore there are 9 data points strictly between 600 and 800. **OR** 19 – (4 + 1 + 4 + 1) = 9	✓ 11 data points from 600 to 800 ✓ 9 data points (2) **OR** ✓ 4 + 1 + 4 + 1 ✓ 9 data points (2) **NOTE:** If a learner uses 9,5 data points from 600 to 800, award 1 mark only.**[8]**

QUESTION 12

12.1.1	500	✓ answer (1)
12.1.2	About 2 050	✓✓ answer (2)
12.1.3	$IQR = Q_3 - Q_1$ $= 2\,300 - 1\,625$ $= 675$	✓ method ✓ answer (2)
12.1.4	240 – 160 = 80 light bulbs	✓ 240 – 160 ✓ answer (2)
12.2	500 – 420 = 80 light bulbs Therefore the cost will be: R5,00 × 80= R400	✓80 ✓R400 (2) **[9]**

TOTAL: 150